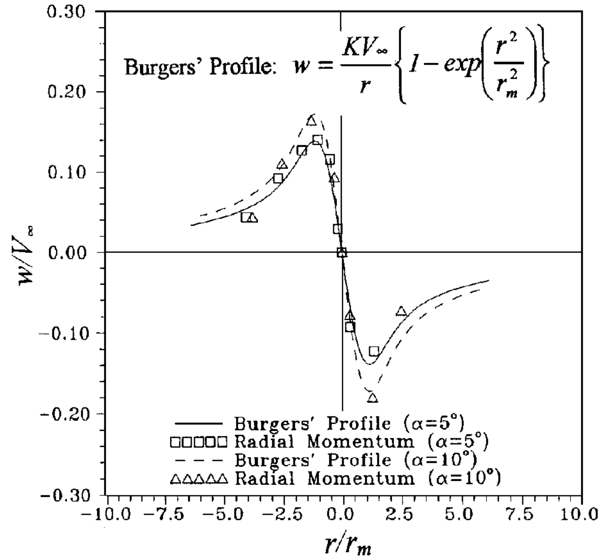

Fig. 4 Vortex swirl velocity profile; K and r_m are the characteristic strength and radius of the Burgers' vortex, respectively.

Conclusion

Results of a near-field wind-tunnel investigation on supersonic tip vortices generated by a straight wing with a sharp chamfered tip have been documented. Time-averaged positions, Mach number, velocity, and thermodynamic properties of the tip vortices were inferred from pitot pressure distributions and shadowgraph flow visualization. The tip vortices have approximately constant axial Mach number, convect downstream with very little radial diffusion of tangential momentum, and have a swirl velocity distribution reasonably described by Burgers' profile. This diffusion suppression quality of the organized structure of the tip vortex may result in the transport of entrained passive contaminants to larger downstream distances than usual wake processes would suggest, which could have significant implications in the design and operation of supersonic transports and/or affect the stealthiness of supersonic designs. A correlation is found to exist between t/c of the wings and the axial Mach number profiles in the cores. It is postulated that chamfering reduces the severity of the crossflow separation pattern around the wing tip, thereby reducing the losses associated with vortex generation. It appears possible then to modify supersonic tip vortices in the near field by simple means such as chamfering the wing tips, a concept that could be valuable in hypervelocity scramjet research where streamwise vortices are being investigated for supersonic mixing augmentation.

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Broadband Vibration Damping Using Highly Distributed Tuned Mass Absorbers

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Introduction

THE constrained layer damping treatment is a popular strategy used to control vibration in aerospace structures. The constrained layer treatment is a strain-based system in the sense that it requires dynamic strain in the base structure to function effectively. In situations where the vibratory motion does not produce significant levels of dynamic base strain, strain-based treatments can be ineffective. Sato et al.[1] examined the effect of both tensile and compressive loads on the transverse vibration of beams with constrained layer damping. The authors demonstrated, both analytically and experimentally, that the modal loss factors could be significantly reduced by in-plane tensile loads.[1] The transverse vibration of pressurized shells and the transverse vibration of helicopter rotor blades are two aerospace examples of dynamic structures that experience substantial in-plane loads in operation.

Unlike strain-based treatments, inertial dampers respond to the motion of the structure, regardless of whether or not that motion produces base strain. The classical inertial damper is a tuned mass absorber, as presented by Nashif et al.[2] Tuned mass absorbers are discrete devices that are typically effective over a very narrow frequency band. The effective band can be broadened somewhat by introducing dissipation into the spring material. The effective band can be widened still further by combining a number of absorbers, each tuned to a slightly different frequency, into a distributed system.

Several authors have examined inertial dampers distributed either in space or in frequency. Nashif et al.[2] examined the effect of distributed dampers on beams, whereas Smith et al.[3] analyzed the transverse vibration of flat plates with distributed tuned absorbers. Igusa and Xu[4] showed, on a single-degree-of-freedom base structure, how multiple tuned mass absorbers tuned to different frequencies could provide broadband energy dissipation.

The analysis of tuned mass systems, distributed both spatially and in frequency is a more recent area of research. In the context of fuzzy structures, Pierce et al.[5] used a distribution of spring/mass systems to model a plate with internal attachments. The authors showed how,

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as the number of absorbers became large, the effective mass of the absorbers could be approximated by a smooth frequency-dependent function.[5]

The present work considers a distributed tuned mass system (distributed spatially and in frequency) as an inertial, broadband damping treatment. The analysis of the damper is based on a numerical approximation to the approach presented by Pierce et al.[5] The effectiveness of the damper is illustrated using a simply supported beam under inplane tension.

Because the analysis of the distributed tuned mass system is similar to the analysis of a beam on an elastic foundation, the present authors refer to the damping treatment as an inertially coupled elastic foundation (ICEF).

Analytical Beam Model

A simply supported beam is used to demonstrate the performance of the ICEF. The beam is isotropic with an in-plane tensile force applied at the neutral axis. Two surface damping treatments are considered, a conventional constrained layer treatment and an ICEF.

The beam configurations are analyzed using an iterative smeared laminate beam model (ISLM) as presented by Zapfe and Lesieutre.[6,7] The ISLM accounts for transverse shear deformation, which is necessary for the analysis of the constrained layer treatment.

Distributed Tuned Mass Absorbers

Consider a system of discrete tuned mass dampers (TMDs), attached to a common rigid base, whose natural frequencies are designed to fall within some frequency band. Motion of the base results in a dynamic reaction force that is exerted by the TMDs. The magnitude and phase of the reaction force are related to the base motion by the effective dynamic mass of the collective system. In the frequency domain, the effective mass $M_d(s)$ can be expressed by

$$M_d(s) = \sum^{N} \frac{m_i}{(m_i/k_i)s^2 + 1} \tag{1}$$

where k_i and m_i are the stiffness and mass of the ith damper. For very low frequencies, the effective mass is simply the collective mass of the N dampers. At high frequency, above the natural frequency of the dampers, the masses are effectively motionless, and the system behaves like N parallel springs. At intermediate frequencies, the effective mass has a magnitude and phase, meaning the damper system has the ability to dissipate energy.

In the limit, as number of dampers becomes large and the spacing between the individual frequencies becomes small, the dampers lose their individuality and the magnitude and phase of the effective mass can be accurately characterized as smoothly varying functions of the driving frequency. The need for large numbers of dampers to achieve a smooth collective behavior can be alleviated through the introduction of dissipation into the spring material.

In the present analysis, energy dissipation is introduced through a complex modulus formulation wherein the spring rate in Eq. (1) is replaced by the complex quantity

$$k_i^* = k_i(1 + j\eta_i) \tag{2}$$

where η_i is the loss factor associated with the spring material.

Substitution of Eq. (2) into Eq. (1), along with the added substitution $s = j\omega$, yields the steady-state effective mass for a collection of inertial dampers,

$$M_d(j\omega) = \sum^{N} \frac{m_i \omega_i^2(1 + j\eta_i)}{\omega_i^2(1 + j\eta_i) - \omega^2} \tag{3}$$

where $\omega_i^2 = k_i/m_i$ is the natural frequency of the ith damper. The spring rate does not appear explicitly in Eq. (3) because it is determined once the mass and natural frequency of the damper are specified. Without dissipation, the magnitude of the effective mass is infinite at each of the damper resonant frequencies. With dissipation, the magnitude and phase of the effective mass are smoothly varying functions over the design band.

Application of the Distributed Damper to a Beam

Although Eq. (3) applies to a system of discrete dampers, it can also describe the ICEF, which has an effective mass per unit length

$$m_d(\omega) = \sum^{N} \frac{\rho_i \omega_i^2(1 + j\eta_i)}{\omega_i^2(1 + j\eta_i) - \omega^2} \tag{4}$$

where ρ_i is the mass per unit length of the ith damper. From an analytical standpoint, the ICEF can be considered as an infinite number of vanishingly small spring/mass dampers. In this limit, the ICEF can be modeled as a homogeneous covering whose mass per unit length happens to be frequency dependent. In reality, the distributed effective mass would only approximate the behavior of a large but finite number of discrete spring/mass dampers. Equation (4) can also describe an effective mass per unit area as would be used in a plate or shell model. In that case, ρ_i would represent the area density of the ith damper.

The effective mass of the ICEF given by Eq. (4) is functionally equivalent to matter that affects the transverse motion of the beam. A tensile force, on the other hand, increases the apparent transverse stiffness of the beam. The effective mass of the ICEF and the effective stiffness of the tensile force are included in the ISLM analysis through the mass and stiffness matrices, specifically through the modification of the $M_{2,2}$ and $K_{2,2}$ terms

$$M_{2,2} = M_1 + M_4 k_n^2 + m_d(\omega), \qquad K_{2,2} = K_4 k_n^4 + T k_n^2 \tag{5}$$

where $k_n = n\pi/L$ is the modal wave number. In the ISLM, M_1, M_4, and K_4 are section properties corresponding to the transverse inertia, rotatory inertia, and bending stiffness, respectively.

Two features of ICEF effective mass should be noted. First, at any driving frequency, m_d is simply a complex number with a specific magnitude and phase. Second, m_d must be evaluated at the correct driving frequency, namely, the modal frequency. Because m_d is essentially a frequency-dependent quantity, the eigenproblem must be solved iteratively. In the present study, the iterative nature of the ISLM beam model naturally accommodates the iterative calculation of m_d.

Numerical Results

A simply supported isotropic beam, subject to an in-plane tensile force, is used to illustrate the benefits of an ICEF over a conventional constrained layer treatment. The beam dimensions and material properties appear in Table 1. The characteristics of the constrained layer treatment were selected to maximize the modal loss factor at 200 Hz. The constrained layer treatment increased the beam mass by 11%.

The modal frequencies and loss factors for the beam with the constrained layer treatment appear in Fig. 1. The modal properties are shown for three tensile loads, 0 N, 1 kN, and 5 kN. The constrained layer treatment produces a peak modal loss factor of $\eta = 0.06$ in the third mode, at 200 Hz. The tensile load has two effects on the modal properties: first, it increases the modal frequencies in much the same way that tension increases the pitch of a guitar string and second, the tension lowers the modal loss factors. The loss factor reduction is most prevalent at low frequency where the tension, which produces

Table 1 Physical properties for numerical example

	Thickness, mm	E, GPa	G, GPa	ρ, kg/m^3	η
Constraining layer	0.3	206.8	82.7	7850	0.0
Damping layer[a]	0.1	$3 * G(\omega)$	$G(\omega)$[b]	2600	$\eta(\omega)$[c]
Base beam	3.0	206.8	82.7	7850	0.0
Beam length = 50 cm, beam width = 5 cm					

[a]Corresponds to 3M ISD112.
[b]$G(\omega) = \exp[-0.003164 \ln(\omega)^3 + 0.072908 \ln(\omega)^2 - 0.041574 \ln(\omega) + 11.877]$.
[c]Loss factor $\eta(\omega) = \exp[-0.003089 \ln(\omega)^3 + 0.052443 \ln(\omega)^2 - 0.29111 \ln(\omega) + 0.41344]$ and ω in radians per second.

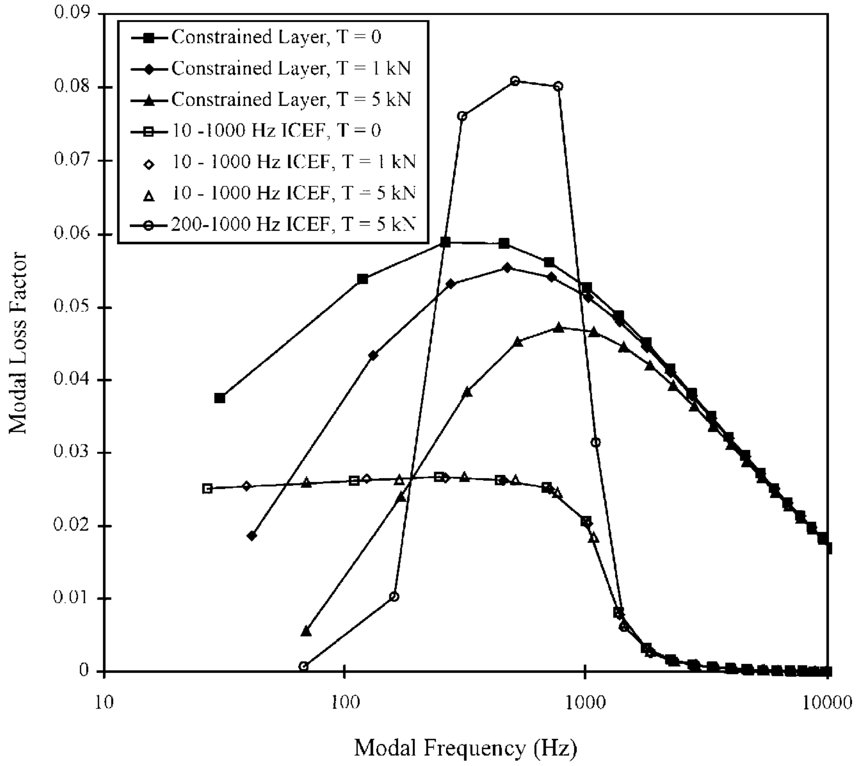

Fig. 1 Modal properties for a simply supported beam treated with a constrained layer damper and an ICEF.

no dynamic strain in the damping layer, accounts for a significant portion of the beam's overall transverse stiffness.

An ICEF was designed to provide energy dissipation over a frequency range of 10–1000 Hz using 20 logarithmically spaced design frequencies. The linear mass density of the damper was 1.18 kg/m, a 10% increase in mass. The mass was equally apportioned to each design frequency. For this design range and frequency spacing, a spring loss factor of $\eta = 0.5$ was required to produce a smoothly varying effective mass without resonant peaks.

The modal properties for the beam treated with the ICEF also appear in Fig. 1. The ICEF produces modal damping primarily over the design range. For frequencies above the design range, the modal damping falls off rapidly to the material loss factor of the base beam. Within the design band, the ICEF produces a uniform modal loss factor of $\eta = 0.026$.

The in-plane tension has no effect on the modal damping of the ICEF. As in the constrained layer example, the modal frequencies increase when tension is applied; however, the modal loss factors are independent of the tensile force. This represents a marked improvement over the constrained layer treatment, the energy dissipating capabilities of which decreased dramatically with increased tensile load.

The uniform modal damping of the ICEF is a consequence of the selection of the mass and frequency distribution. By adjusting the design frequencies, dissipation, and mass distribution, the ICEF can be tailored to address specific frequency bands. To illustrate this, an ICEF with a design range of 200–1000 Hz was applied to the beam. In this case, because of the narrower design range, a spring loss factor of $\eta = 0.2$ was sufficient to produce a smoothly varying effective mass. The modal data for this example also appear in Fig. 1. The narrower design band substantially increases modal damping. The peak modal loss factor of $\eta = 0.08$ under tension actually surpasses the peak performance of the unloaded constrained layer treatment.

Interestingly, the effect of the spring loss factor on the ICEF generated modal loss factors runs contrary to intuition. An increase in the spring loss factor, beyond the point necessary for a smooth effective mass, does not increase the peak modal damping. An increased spring loss factor widens the damper's effective bandwidth at the expense of the damper's peak performance. For example, the 200–1000 Hz ICEF with a spring loss factor of $\eta = 0.3$ actually lowers the peak modal loss factor by 5%.

Conclusions

The effectiveness of strain-based damping treatments may be significantly affected by the presence of in-plane tensile loads, which can lead to transverse vibration modes that involve very little dynamic strain in the base structure. Inertial dampers, by contrast, respond to the motion of the base structure, irrespective of the mechanism responsible for the motion. Using a large number of inertial absorbers distributed in space and in frequency, a broadband damper can be designed to provide energy dissipation over a specific frequency band that is immune to the adverse effects of inplane tensile loads.

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